## Scientific Report: STSM in INESC TEC Porto for KEP Cost Action

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October 28, 2018

I have visited the Operational Research group from the INESC TEC institute of Porto in order to discuss how to approach and solve mathematical problems of stochastic optimisation for the choice of pairing and transplants in Kidney Exchange Programs, between the 19th and 25th of October 2018.

The group in Porto is involved in a project on Kidney Exchange Programs with multiple programs (mKEP), funded by the Portuguese Science Fundation. During my stay, we held several discussions about two problems linked to KEP and mKEP problems. The basis of KEP problems is to find the optimal pairings between patients and donors, which mathematically translates into finding a set of cycles inside a directed graph where patient-donor pairs are represented by nodes and their respective potential compatibilities by arcs. Most of the time these compatibilities are estimated based on simple characteristics like blood type. When the actual transplants are decided, additional compatibility tests are performed and some transplant cycles have to be abandoned or modified to cope with it. Our attempts aim at designing and solving models where the possibility of such compatibility failings are taken into account when choosing the patients and donors for transplants, based on estimated probabilities of failure for compatibilities or patients and donors pulling out of the program for various reasons. We discussed two possibilities in this context: one regarding the choice of cycles involving patients from different KEP programs cooperating together, as the success of these transplants is critical for the survival of such international cooperations; and another regarding the possibility to use some budget (be it financial and/or time) to probe more efficiently a few potential compatibilities in the graph before deciding for the transplants and surgery, in order to reduce the risks of transplant cycles.

## 1 Stochastic models for mKEP

The group of INESC TEC is already working on a model to ensure fairness between several national KEPs cooperating together and joining their patient-donor pools. We would however like extend such work by taking into account the possibilities of failure of the proposed transplants, particularly those involving patients and donors from different pools (i.e. countries), since the success of the international transplants is a key to the long term success of the multi Kidney Exchange Programs. During our discussions in Porto, I have proposed to use recent methodologies of scenario aggregation for solving a stochastic mKEP model that uses a measure of robustness to assess the reliability of the transplant pairings between different coutries, called Conditional Value at Risk (CVaR). This type of Stochastic Integer Problem is usually hard to solve without making some statistical approximations which make the results of the optimisation algorithm less reliable. However, generalising results of e.g. Song and Luedtke (SIAM Journal on Optimization, 2015), I believe we can manage to solve such a model exactly and propose valid and robust transplant proposition for deciders of multiple KEP cooperations. A new postdoc member of the group in Porto will be in charge of implementing the stochastic algorithm after we finish deriving the necessary stochastic linear equations. We will use the instance generator of the group which has proven its ability to furnish realistic KEP instances. We will also use probabilities of failure for the donor-patient compatibilities which are more realistic than those used in the literature on stochastic KEP so far (Klimentova et al 2016, Zheng et al 2015, etc...), based on PRA values generated for the patients. This will further increase the consistency of KEP instances regarding the number of compatibilities of a patient and the probabilities of failure of such compatibilities.

We hope to obtain a complete model within a few weeks at which point the implementation and testing of the optimisation algorithm will begin during the following months. We are confident that such an approach will provide interesting results and that our proposed methodology is also of interest to other stochastic optimisation problems, therefore we have high hopes that our future results can be published in an international scientific journal.

## 2 Probing compatibilities for stochastic KEP models

Additionally to the above optimisation problem, we would like to explore another avenue of research for stochastic KEP problems with transplant failures. Since the compatibilities are a priori decided based on basic riteria, the more thorough testing made when transplants are decided can indicate a failure of the transplantation plan. In some KEPs where the pool is small enough, all possible compatibilities are extensively checked to ensure that no bad surprise arises after choosing the pairs for transplantation. For bigger pools this can prove impossible due for example to time or cost restrictions. We propose to allow for the exact probing a limited number of compatibilities in the pool before deciding for the transplants to implement, in order to reduce the uncertainty before taking important decisions.

The group in INESC TEC has already started investigating this possibility with approximate algorithms. My proposal to the group was to formalise the problem into a rigorous Stochastic Integer Program and try to tackle the model with an exact algorithm, providing us with the optimal, or close to optimal solution. This stochastic model is extremely challenging to solve because of its mathematical characteristics. However, some methods can be used to solve it approximately but within a given, controlled precision with respect to the optimal objective value (i.e. the expected number of transplants). The method I am advocating will use as a basis the scenario aggregation techniques desribed in the previous section, with additional branching techniques to take into account the additional binary variables of the problem.

The problem is again interesting both from the application and the formal perspective, since our approach could be of interest to Operational Research practitioners working on stochastic problems with similar characteristics to ours. The implementation of the optimisation algorithm will use the basic structure of the algorithm presented in the previous section, creating some kind of synergy between both approaches on the technical level. I therefore estimate that this project can be carried out in a few months after completion of the program advocated in the first section and will probably require itself a few more months on implementation, along with the testing of several different branching strategies and approximation methods to be coupled to our exact stochastic framework. This line of research is more exploratory than the previous one, however if no unexpected surprises arise during the development and implementation of our model, we expect to obtain publishable results for the second part of next year (roughly one year from now), in the form of an article in a scientific international journal. The sophistication of the optimisation algorithm along with the direct applicability of the model would probably warrant publication in a excellent journal of Operational Research.